Re-Entry Vehicle Base Pressure and Heat-Transfer Measurements at $M_\infty = 18$

Bruce M. Bulmer*
Sandia Laboratories, Albuquerque, N.M.

Nomenclature

h = static enthalpy
H = total enthalpy
M = Mach number
Nu = Nusselt number
p = static pressure
Pr = Prandtl number
$\dot{q}$ = heat flux
r = base coordinate measured from base outer edge
R = base coordinate measured from base centerline
R_B, R_N = base and nose radius, respectively
R_N/R_B = bluntness ratio
Re = Reynolds number
St = Stanton number
V = velocity
γ = ratio of specific heats
θ_c = cone half-angle
ρ = static density

Subscripts

b = base or outer base condition
e = local cone (boundary-layer edge) condition
L = based on axial length of cone
s_L = based on wetted length of cone
w = wall condition
∞ = freestream condition

Introduction

Hypersonic near-wake studies often utilize base pressure[1-4] and base heat-transfer[5,6] data derived from full-scale flight tests to compare with analytical and experimental results. Base pressure estimates are important, for example, in near-wake flowfield analyses and drag predictions, while heat-transfer calculations are necessary to define afterbody thermal protection requirements. In this Note, additional flight-test base pressure and heat-transfer results are presented for a slender cone; data are limited to pressure measurements in laminar flow and heat-transfer measurements in turbulent flow conditions. Data analyses include correlation with flight-test data and comparison with near-wake theory. Comparisons of the present results with published data correlations emphasize the Reynolds-number dependence of laminar base pressure and the various factors that influence turbulent base heating in hypersonic flow.

Re-Entry Vehicle and Instrumentation

The flight configuration was a slender flat-based cone with $\theta_c = 9°$ and $R_N/R_B = 0.061$. For that portion of the flight for which laminar data are presented, the Mach number was essentially constant at $M_\infty \approx 18$; the Mach number was, in general, variable during the turbulent portion of the flight. Freestream conditions were based on the calculated post-flight trajectory, and the computed local cone conditions (at the boundary-layer edge on the frustum immediately preceding the base) included nose-bluntness and real-gas effects.

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Index categories: Entry Vehicle Testing; LV/M Aerodynamic Heating; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

* Member, Technical Staff, Re-Entry Vehicle Aerothermodynamics Division. Member AIAA.

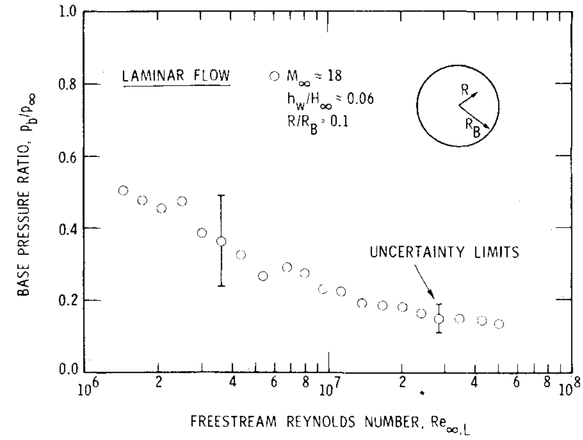

Fig. 1 Base pressure data.

Base instrumentation included a pressure transducer located near the base centerline ($R/R_B = 0.1$) and a calorimeter near the edge of the base ($R/R_B = 0.8$). The pressure transducer was a variable-reluctance type with 0–1 psia range output telemetered at 15 samples/sec. Pressure time lag effects were neglected because a short pneumatic system (tube length/port diam. ratio of 40) was used. The asymptotic calorimeter had 0–30 Btu/ft^2—sec range output telemetered at 15 samples/sec and was mounted flush with the base cover surface to insure data accuracy.

Results and Discussion

Both pressure and heat-transfer data are presented in terms of Reynolds number in Figs. 1 and 2. Measured base pressures are normalized by freestream static pressure, and the base heat-flux data are nondimensionalized as Stanton number. Uncertainty limits are indicated for each set of data. The base pressure results correspond to absolute pressure levels exceeding 2% of the full-scale transducer output, and the heat-transfer data exceed 10% of full scale.

Base pressure analysis

Base pressure data in Fig. 1 reveal the expected dependence on Reynolds number for $Re_{\infty,L} = 1 \times 10^6$ to 5×10^7 in laminar flow. Comparison with flight-test data for slender cones from Cassanto,[1] Ohrenberger and Baum,[2] and Batt[3] (Fig. 3) indicates a consistent variation with Reynolds number for $M_\infty = 18$–21, despite differences in external configuration and base radial location. This data correlation supports Batt's conclusion that p_b/p_∞ and $Re_{\infty,L}$ represent proper scaling parameters for slender

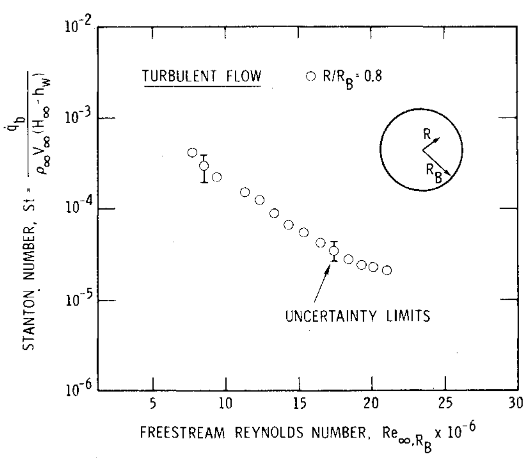

Fig. 2 Base heat-transfer data.

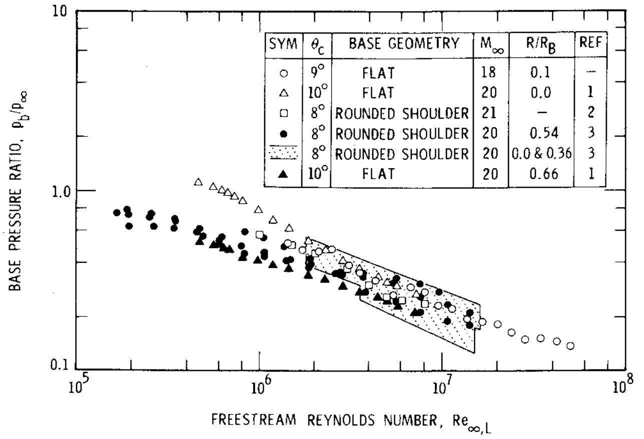

Fig. 3 Laminar hypersonic flight-test base pressure data for slender cones.

hypersonic cones. The radial base pressure gradient is also evident in Fig. 3; at low Reynolds numbers, the data level for the centerline ($R/R_B = 0.0$ and 0.1) is generally above that for $R/R_B = 0.54$ and 0.66, while the data band for $R/R_B = 0.0$ and 0.36 overlaps both regions.

In Figs. 4 and 5, the present data are compared with base pressure correlations and theoretical near-wake results. Fair agreement (particularly in slope) is observed in Fig. 4 with a previous data correlation[4] using local parameters from the Reeves and Buss theory.[7] An additional comparison of the Reynolds-number slope is provided (Fig. 5) with numerical solutions from Ohrenberger and Baum[2,8] for a cold-wall 8° cone at $M_\infty = 21$.† The slope indicated by the present data is in close agreement with these theoretical results and with the -0.4 slope derived from Fig. 4 for a sharp cone at constant M_∞.

Base heat-transfer analysis

Heat-transfer data in Fig. 2 correspond to fully turbulent conditions over the vehicle (turbulent flow was verified by various onboard thermal instrumentation). These data indicate large increases in Stanton number with decreasing Reynolds number and reflect the change in the base pressure ratio with decreasing Mach number.[6]

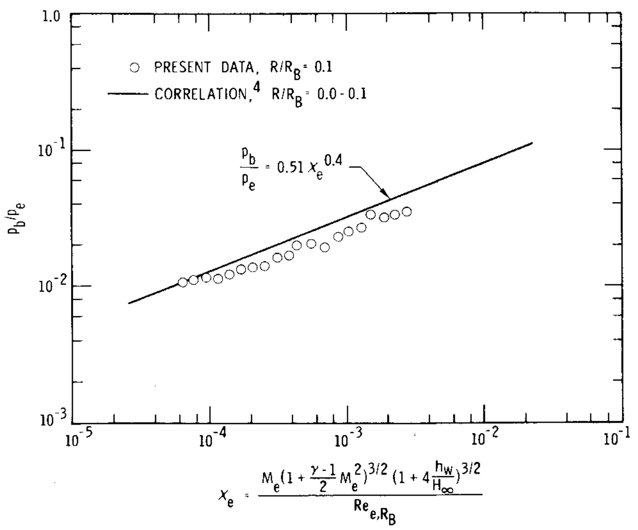

Fig. 4 Comparison of base pressure data and correlation using local parameters from the Reeves and Buss theory.

† Ideal gas solutions[8] (dash line), depicting the rate of change of p_b/p_∞ with $Re_{\infty,L}$, were shifted at constant slope to correspond with the near-wake solution[2] at $Re_{\infty,L} = 4 \times 10^6$.

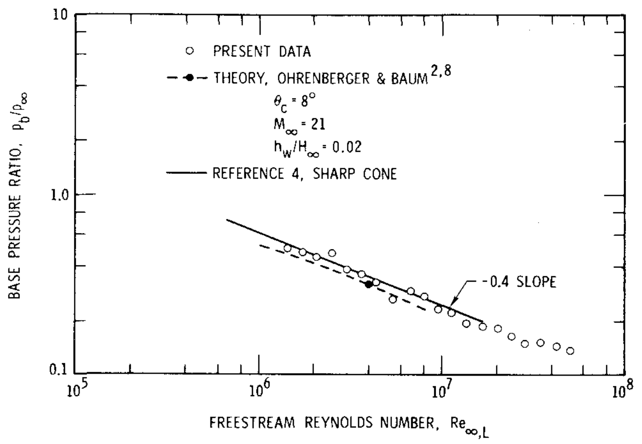

Fig. 5 Comparison of base pressure data and near-wake theory.

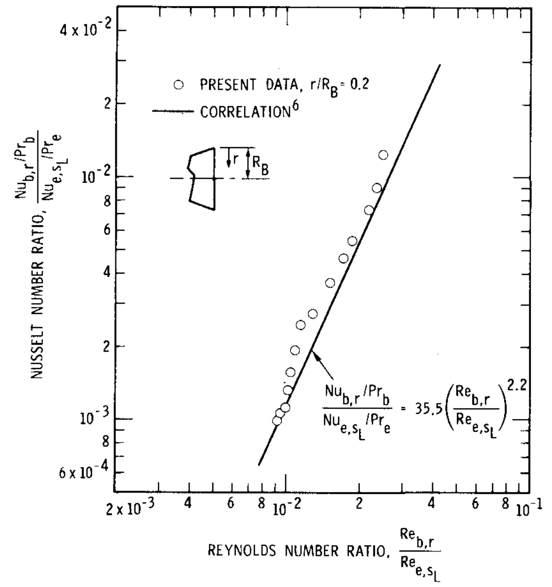

Fig. 6 Comparison of base heat-transfer data and correlation.

For the present analysis, the data were reduced in terms of the Nusselt and Reynolds number ratios described in Ref. 6. The outer base flow properties were evaluated outside of the dividing streamline by expanding the local cone conditions to the estimated base pressure at the calorimeter location (determined from a turbulent base pressure correlation at $R/R_B = 0.8$ for identical vehicles).

Results of this analysis are compared with the previous heat-transfer correlation in Fig. 6. Note that the base Nusselt and Reynolds numbers include the radial base heat-transfer gradient[5,6,9] by incorporating the actual calorimeter location measured from the base outer edge. The present data clearly reveal the influence of both the radial gradient and the local cone flowfield on turbulent base heat transfer. In addition, the large increase in the Nusselt number ratio represented by these data is indicative of the base pressure variation with Mach number which affects the Reynolds number ratio directly.

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Quasicoordinate Equations for Flexible Spacecraft

PETER W. LIKINS*
University of California, Los Angeles, Calif.

Introduction

THE purpose of this Note is to quash the quasicoordinate controversy. For those with the responsibility for developing simulation capabilities for spacecraft of increasing flexibility and dynamic complexity, with increasingly stringent pointing requirements, and with the increasing importance of cost effectiveness calling for dramatic improvements in computational efficiency, the stories that circulate in technical discussions over the promise of the quasicoordinate approach are seductive indeed, and worthy of investigation. This Note describes the results of one such investigation, which culminated in the following proposition.

Proposition

Applying Euler's rotational equation in the vector form $\mathbf{M} = \dot{\mathbf{H}}$ *to any material continuum, where* $\mathbf{M}$ *is the moment of external forces about the system mass center and* $\dot{\mathbf{H}}$ *is the inertial time derivative of the system angular momentum about the system mass center, and recording scalar equations for an orthogonal vector basis fixed in any reference frame* f *in which the system mass center is fixed, including the inertial angular velocity* $\boldsymbol{\omega}$ *of* f *among the variables, produces the same three equations of motion that emerge from Lagrange's quasicoordinate formulation, with the scalar components of* $\boldsymbol{\omega}$ *for a vector basis fixed in* f *chosen as the quasicoordinate derivatives.*

The validity of this proposition in application to a single rigid body is well known,[1,2] but its general applicability does not appear to have been demonstrated. The importance of this proposition lies not in what it tells us to do in order to improve our spacecraft simulation programs, but in what it tells us not to do; if we are already in possession of a system of equatons or a simulation program based on a Newton-Euler formulation, we should *not* make the investment required to obtain a new set of equations or a new computer program based on a quasicoordinate formulation, and conversely.

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* Professor. Associate Fellow AIAA.

Theoretical Background

Although Whittaker[1] tells us that particular cases of the quasicoordinate equations were known to Lagrange and Euler, and that the general form is due to Boltzmann (1902) and Hamel (1904), still the use of quasicoordinates is not widespread. The so-called Lagrangian quasicoordinate equations† are included in recent books,[2] and modern variants of the quasicoordinate equations have been advanced,[3] but still the concept remains on the edge of memory for most dynamicists, and genuinely familiar to very few. The only application of quasicoordinates to flexible spacecraft in the literature is the interesting work of Bodley and Park,[4] in which Lagrange's quasicoordinate equations are foregone in favor of a direct D'Alembert approach employing quasicoordinates.

Lagrange's equations for generalized coordinates $q_1, \ldots, q_v$, appear in their most general form as the matrix equations

$$(d/dt)(T_{,\dot{q}}) - T_{,q} = Q - A^T\lambda \tag{1}$$

in which T is the kinetic energy expressed in terms of the scalars $q_1, \ldots, q_v$ in the column matrix q and the scalars $\dot{q}_1, \ldots, \dot{q}_v$ in the column matrix $\dot{q}$; the comma convention is used for partial differentiation (so that $T_{,q}$ and $T_{,\dot{q}}$ are $v \times 1$ matrices); Q is the $v \times 1$ matrix of generalized forces defined by

$$Q_k \triangleq \int \dot{\mathbf{R}}_{,\dot{q}_k} \cdot d\mathbf{f}, \qquad k = 1, \ldots, v \tag{2}$$

where $\dot{\mathbf{R}}$ is the inertial velocity of a differential element subjected to force $d\mathbf{f}$; λ is an $m \times 1$ matrix of Lagrange multipliers, and A is an $m \times v$ matrix appearing in constraint equations having the *simple* or *Pfaffian* form

$$A\dot{q} + B = 0 \tag{3}$$

for some $A(q, t)$ and $B(q, t)$. Equations (1) and (3) comprise a complete set of equations, but they are restricted in that they are formulated in terms of generalized coordinates, which by definition comprise a set of scalars the full knowledge of which is sufficient to establish the complete state of the dynamical system as a function of time. Equations of motion may be less complex in form and more readily solved when expressed in terms of quantities representing linear combinations of generalized velocities, such as the scalars $u_1, \ldots, u_v$ in the matrix equation

$$u = W^T\dot{q} + w \tag{4}$$

where W and w may depend upon $q_1, \ldots, q_v$ and t. The scalars $u_1, \ldots, u_v$ may not be derivatives of generalized coordinates; they are sometimes[2] called derivatives of *quasicoordinates*. After transposition of Eq. (4), it becomes apparent that W may be expressed as

$$W = u_{,\dot{q}}{}^T \tag{5}$$

Equations of motion equivalent to Eq. (1) can be formulated in quasicoordinate form as

$$\frac{d}{dt}(\bar{T}_{,u}) + W^{-1}[(u^T - w^T)W^{-1}W_{ij,q}]\bar{T}_{,u} + (W_{,t} - w_{,q}{}^T)\bar{T}_{,u} - W^{-1}\{(u^T - w^T)W^{-1}W_{,q_k}\bar{T}_{,u}\} - W^{-1}\bar{T}_{,q} = W^{-1}(Q - A^T\lambda) \tag{6}$$

with the following notational conventions: $\bar{T}$ is the kinetic energy expressed in terms of u and q; the expression within braces is the kth element of a $v \times 1$ column matrix; the expression within square brackets is the element of a square ($v \times v$) matrix in the ith row and jth column. Equation (6) is somewhat more general than the form of Lagrange's quasicoordinate equations normally encountered,[1,2] although its proof is straightforward.[5] This level of generality is uninspiring for obvious reasons; unless some simplification is introduced to eliminate the necessity of inverting W either numerically at each integration step or literally in advance of integration [in addition to the inversion necessitated by time-varying coefficients of u arising from $d(\bar{T}_{,u})/dt$], Eq. (6) is going to be even uglier than Eq. (1).

The most obvious specialization of Eq. (6) results from replacing Eq. (4) by $u = \dot{q}$; then Eq. (6) reduces to Eq. (1). A more useful specialization of Eq. (6) arises when Eq. (4) is replaced by

† Also known as the Boltzmann-Hamel equations.